Approval: 8th Senate Meeting

Course Name: Numerical Methods in Quantitative Finance

Course Number: MA565

Credit: 3-0-0-3

Prerequisites: IC 110: Engineering Mathematics, IC 111: Linear Algebra

Intended for: UG/PG

Distribution: Elective

Semester: Odd /Even

Course Preamble: Finite difference theory has a long history and has been applied for more than 200 years to approximate the solutions of partial differential equations in the physical sciences and engineering. It is notable that the behaviour of a stock (or some other underlying) can be described by a stochastic differential equation. Then, a contingent claim that depends on the underlying is modelled by a partial differential equation in combination with some initial and boundary conditions. Solving this problem means that we have found the value for the contingent claim. Due to the complexity of these models it is very difficult to find exact or closed solutions for the pricing functions. Even if a closed solution can be found it may be very difficult to compute. For this and other reasons we need to resort to approximate methods. Our interest in this course lies in the application of the finite difference method (FDM) to these problems.

Course Outline: The goal of this course is to develop robust, accurate and efficient numerical methods to price a number of derivative products in quantitative finance. We focus on one-factor and multi-factor models for a wide range of derivative products such as options and fixed income products. This course covers numerical methods relevant to solving the partial differential equations of mathematical finance. Theoretical and practical issues are treated. Topics include (but are not limited to): background material in ordinary and partial differential equations, examples of exact solutions including Black Scholes and its relatives, finite difference methods including algorithms and question of stability and convergence, and a brief introduction to numerical methods for solving multi-factor models.

This course is an introduction to FDM and how to use it to approximate the various kinds of partial differential equations in financial engineering.

Modules:

UNIT 1: [**8 Lectures**]

THE CONTINUOUS THEORY OF PARTIAL DIFFERENTIAL EQUATIONS -- An Introduction to Ordinary Differential Equations, An Introduction to Partial Differential

Equations, Second-Order Parabolic Differential Equations, An Introduction to the Heat Equation in One Dimension, An Introduction to the Method of Characteristics.

UNIT 2: [**8 Lectures**]

FINITE DIFFERENCE METHODS: THE FUNDAMENTALS – An Introduction to the Finite Difference Method, An Introduction to the Method of Lines, General Theory of the Finite Difference Method, Finite Difference Schemes for First-Order Partial Differential Equations, FDM for the One-Dimensional Convection–Diffusion Equation, Exponentially Fitted Finite Difference Schemes.

UNIT 3: [**8 Lectures**]

APPLYING FDM TO ONE-FACTOR INSTRUMENT PRICING - Exact Solutions and Explicit Finite Difference Method for One-Factor Models, Exponentially Fitted Difference Schemes for Barrier Options, Advanced Issues in Barrier and Lookback Option Modelling.

UNIT 4: [**8 Lectures**]

FDM FOR MULTIDIMENSIONAL PROBLEMS – Finite Difference Schemes for Multidimensional Problems, Operator Splitting Methods: Fractional Steps, ADI Methods.

UNIT 5: [**10 Lectures**]

APPLYING FDM TO MULTI-FACTOR INSTRUMENT PRICING - Options with Stochastic Volatility: The Heston Model, Finite Difference Methods for Asian Options and Other 'Mixed' Problems.

Text books:

1. Daniel J. Duffy. *Finite Difference Methods in Financial Engineering: A Partial Differential Equation Approach*, John Wiley & Sons Ltd., 2006.
2. D. Tavella and C. Randall. *Pricing Financial Instruments: The Finite Difference Method*, Wiley, ISBN: 978-0-471-19760-7.

Reference Books:

1. Paolo Brandimarte. *Numerical Methods in Finance and Economics: A MATLAB-Based Introduction*. John Wiley & Sons INC., 2nd Edn., 2006.
2. John A. D. Appleby, David C. Edelman, John J. H. Miller. *Numerical Methods for Finance,* Taylor & Francis, 2008.
3. Michele Breton, Haten Ben-Ameur. *Numerical Methods in Finance*, Springer, 2005.